Math 690.101 Fall 2017 CRN 3218

Special Topics:
Hilbert's 10th Problem

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Marshall University

Marshall University Syllabus

Course Title/Number	**Math 690: Special Topics**	**CRN 3218**
Semester/Year	Fall 2017	
Days/Time	Tuesday and Thursday 12:30pm – 1:45pm	
Location	WAEC 3119	
Instructor	Carl Mummert	
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Office	Morrow Library 110	
Office Hours	Tuesday and Thursday: 10:00am – 11:30am Monday and Wednesday: 10:00am – 11:30am	
University Policies	By enrolling in this course, you agree to the University Policies listed below. Please read the full text of each policy by going to www.marshall.edu/academic-affairs and clicking on "Marshall University Policies." Or, you can access the policies directly by going to **www.marshall.edu/academic-affairs/policies/** *Policies:* Academic Dishonesty / Excused Absence Policy for Undergraduates / Computing Services Acceptable Use / Inclement Weather / Students with Disabilities / Academic Dismissal / Academic Probation and Suspension / Academic Rights and Responsibilities of Students / Affirmative Action / Sexual Harassment.	

Course Description

In 1900, David Hilbert posed a problem: devise an algorithm to determine which polynomial equations can be solved with rational numbers. Seventy years later, building on the work of Alan Turing in computability theory, a group of mathematicians showed that Hilbert's request is impossible to fulfill.

In this class, we will look at the mathematics and the history of Hilbert's 10th problem, using original writings of Hilbert, Turing, Davis, Putnam, Robinson, and Matiyasevich. The course is for students interested in mathematics, computer science, and the relationship between these fields.

Prerequisite

MTH 220 or 230 with a grade of C or higher.

Course goals – what will you learn?

At the end of this course:

1. Recall and apply the key theorems and definitions related to Hilbert's 10th problem.
2. Recall and explain examples, compare them with each other, and apply them to produce counterexamples.
3. Write proofs to verify the correctness of propositions related to the course material.
4. Write proofs the demonstrate a level of mathematical correctness and precision appropriate for an undergraduate mathematics major.
5. Use LaTeX software to produce mathematical documents.
6. Recall and apply the fundamental notions of computability theory.
7. Summarize the history of Hilbert's 10th problem and the mathematicians involved.

For more information see "Learning Outcomes" below.

Assignments – what do you have to do?

There are several kinds of assignments in this course:

- *Exams* (20% of grade): There will be two exams during the semester. Each of these is worth 15% of your grade.
- *Poster* (20% of grade): The class will produce two posters: one for the mathematicians behind the MDRP theorem, and one on the problem itself. Each poster will have a team of students. These posters will take the place of the final exam.
- *Homework and Quizzes* (25% of grade): There will be a written homework assignments due throughout the semester. Some of these will be computation-based, some will be proof-based, and some will use mathematical software. I may also give in-class quizzes, which will always be announced in advance. These will count the same as homework assignments.
- *Class notes* (25% of grade): The class will collaboratively write a set of notes based on the topics discussed when we meet. Each day, one student will be responsible for keeping notes and placing them into a shared LaTeX document on Overleaf. The grade on this assignment is based on the quality and timeliness of each submission; see below.
- *Final oral exam* (10% of grade): You will take an oral exam during final exam week on the material of the class. A list of topics will be distributed in advance.

Exam dates

- Exam 1: Thursday, October 12
- Exam 2: Thursday, November 9
- Final oral exam: individually scheduled during final exam week

Grading scale

Your grade in the course will be assigned on the following scale:

- 90–100: A
- 80–90: B
- 70–80: C
- 60–70: D

Attendance policy

You should attend each class, with the same standards for absences you would use for a professional job. I will record attendance each day. If you need to be absent from class, you should contact me promptly – before the absence, except in case of emergency, just as with a professional job. I will excuse absences that are covered by the university's excused absence policy. I may ask for documentation in cases of repeated absences, or for absences on exams.

Because there are numerous weekly homework and quiz assignments, there will be no make-ups on these. Excused assignments will not count towards your grade.

Make-up exams will always require documentation for the absence, and will be scheduled promptly after the exam.

Anti-plagiarism policy

Plagiarism of any kind is not permitted. Students who plagiarize on an assignment will receive a zero for that assignment, and the university-wide plagiarism policy will be followed. If you are unsure about what is permitted on a particular assignment, please ask well before the due date.

Course Schedule

In general, the class will have three phases.

1. Introduction to Hilbert's 10th problem and its mathematical setup
2. Introduction to computability theory
3. Solution of Hilbert's 10th problem: MDRP theorem

Daily Schedule

Aug 22 Introduction, LaTeX	Aug 24 Hilbert's problems
Aug 29 Diophantine sets	Aug 31 Diophantine sets
Sep 5 Diophantine sets	Sep 7 Diophantine sets
Sep 12 Computability	Sep 14 Computability
Sep 19 Computability	Sep 21 Computability
Sep 26 Computability	Sep 28 Computability
Oct 3 Hilbert's 10th	Oct 5 Hilbert's 10th
Oct 10 Discussion	Oct 12 **Exam 1**
Oct 17 Hilbert's 10th	Oct 19 Hilbert's 10th
Oct 24 Hilbert's 10th	Oct 26 Hilbert's 10th
Oct 31 Hilbert's 10th	Nov 2 Hilbert's 10th
Nov 7 **Exam 2**	Nov 9 J. Robinson
Nov 14 Poster workday	Nov 16 Hilbert's 10th
Nov 28 Hilbert's 10th	Nov 30 Hilbert's 10th
Dec 5 Generalizations	Dec 7 Discussion

Learning outcomes

The table below shows the learning outcomes for the course. They describe the main skills that you will be tested on in the course. In general, the assignments that are intended to "practice" a skill will be graded with more partial credit than assignments that are intended to "assess" a skill.

Learning outcome	How the outcome will be practiced	How the outcome will be assessed
Recall and apply the key theorems and definitions related to Hilbert's 10th problem.	Homework, In-class activities	Exams
Recall and explain examples, compare them with each other, and apply them to produce counterexamples.	Homework In-class activities	Exams
Write proofs to verify the correctness of propositions related to the course material.	Homework, In-class activities	Exams, Course notes
Write proofs the demonstrate a level of mathematical correctness and precision appropriate for an undergraduate mathematics major.	Homework, In-class activities	Exams, Course notes
Use LaTeX software to produce mathematical documents.	In-class discussion	Course notes, Poster
Recall and apply the fundamental notions of computability theory.	Homework, In-class activities	Exams
Summarize the history of Hilbert's 10th problem and the mathematicians involved.	In-class activities,	Exams, Poster

Proof grading rubric

	Mastery	**Developing**	**Beginning**	**Rudimentary**
Mathematical writing	All variables are properly introduced before they are used. The use of quantifiers is clear. Symbols and terminology are used appropriately. The solution is written in polished prose.	Some variables are used without being introduced. Symbols and terminology are used appropriately. The solution is written in prose.	Some variables are used without being introduced. Some symbols or terminology are used incorrectly. The bulk of the solution is written in prose.	Some variable are written without being introduced. Symbols are used inappropriately. Some terminology is used incorrectly. The solution is not written in prose form.
Logical reasoning	The logical reasoning is correct and clearly explained. The solution is complete: all cases have been examined, all significant steps have been justified, and all assumptions have been clearly stated. The solution is clearly organized and the argument is easy to follow.	The logical reasoning is essentially correct, although some parts are not clearly explained. Only minimal revision would be needed to correct the explanation. All cases have been examined, all significant steps have been justified, and all assumptions have been clearly stated. The solution is organized well enough that the structure of the argument is clear.	The logical reasoning has a minor flaw, which requires rewriting part of the argument. The solution is not complete: some case has not been examined, a significant step has not been justified, or an unspoken assumption has been made. Some parts of the solution are not clearly explained. The organization makes it difficult to discern the structure of the argument.	The logical reasoning has a serious flaw or multiple minor flaws. Significant revision is required to correct the solution. The solution is not complete: some case has not been examined, a significant step has not been justified, or an unspoken assumption has been made. Some parts of the solution are not clearly explained. The solution is not well organized.
Surface features	The problem is clearly stated. Grammar and spelling errors are rare. The formatting matches the submission guideline.	The problem is clearly stated. Grammar and spelling errors do not distract from the content. The formatting matches the submission guideline.	The problem is clearly stated. Grammar and/or spelling errors distract from the content. Formatting does not meet the submission guideline.	The statement of the problem is missing or unclear. Grammar and/or spelling errors distract from the content. The formatting does not meet the submission guideline.