Advanced Differential Equations

Spring 2015

MTH 416 Section 201, CRN 4167

**T,R 11:00-12:15 SH 511**

(Updated 1/22/2015)

# Instructor: Dr. Bonita A. Lawrence

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Office Hours: 2:00 P.M. – 3:30 P.M. T,R

10:00 A.M. – 11:30 A.M. M,W

Or a time that we can find that

works for both of us!

**General University**

**Policies:** By enrolling in this course, you agree to the University Policies presented below. You can read the full text of these important policies online using the following path: Marshall Home Page - Course Catalogs – Undergraduate Catalogs. At this point, choose the catalog you started under (or any catalog after that).

**University Attendance**

**Policy**: The University Policy that describes excused absences can be found in the Marshall University 2013 – 2014 Undergraduate Catalog on pages 83 – 84.

**Academic Dishonesty Policy:** I expect you to do your own work. You can certainly discuss the homework problems with your colleagues but what you present to me for any type of assessment must be your own. The University’s policy concerning academic dishonesty can be found in the Marshall University 2013 -2014 Undergraduate Catalog on pages 69 – 70.

**Policy for Students with Disabilities:** Marshall University is committed to equal opportunity for all. Students with physical, learning or psychological disabilities should contact the Office of Disabled Students Services (DSS) in Prichard Hall Room 117, 304 696-2271 and provide documentation of their disability. After consultation the DSS coordinator will send a letter to the student’s instructors describing the accommodations the student will need. For more information, go to <http://www.marshall.edu/disabled> or call or visit the office in Prichard Hall.

**Affirmative Action Policy:** In the spirit of equal opportunity for all, Marshall University has an Affirmative Action Policy. This can be found in the Marshall University 2013 - 2014 Undergraduate Catalog on p. 66.

**Inclement Weather Policy:** In the event of bad weather that may prevent us from coming to school, Marshall has a policy that describes how things will be handled. (Note that I have been here for 12 years and we have only shut down school one day during this time.) The policy can be found on pp. 67 -68 of the Marshall University 2013 – 2014 Undergraduate Catalog.

**Course Description from Catalog:** Differential equations are studied qualitatively. Topics include the existence and uniqueness of solutions and the behavior of solutions including stability of nonlinear systems, periodic solutions, and approximation using perturbation methods. (PR: *C* or better in MTH 330 and *C* or better in MTH 335)

**Course Prerequisites:** There is no formal prerequisite for this course. I will assume that you have had the first course in advanced calculus and a proof techniques course. If this is not the case, please come to talk to me about your previous training.

**Course Objectives:** This course is a qualitative study of these lovely mathematical equations that offers us a characterization of unknown function. We will use the nature of the expression and classical results that have been developed through the centuries to describe the behavior of the unknown function under certain conditions. The standard undergraduate course in differential equations presents methods for solving various types of DE’s. This semester we will focus more on whether or not a solution actually exists and how it behaves. We will study the qualitative behavior of solutions, including topics such as stability of nonlinear systems, existence of periodic solutions, and finding approximations using perturbation methods.

Success in the course will be measured by your ability to meet the following learning outcomes.

The ability to

1. Apply the results found in classical theorems (and the proofs of the theorems) in differential equation theory.

Skill Development: Individual, small group and whole group analysis of classical theorems and their proofs. Topics under consideration include existence and uniqueness of solutions, stability and bifurcations. This includes homework, boardwork and Excursions. Exercises will be assigned daily and followed up by timely feedback.

Assessment: Evaluation of written and oral presentations for proper applications of the results of classical theorems and proper uses of the methods used to prove these theorems.

1. Expand understanding of qualitative properties of first order differential equations to higher order systems of differential equations.

Skill Development: Individual, small group and whole group development of the structure required to study higher order systems of differential equations, including classical theorems, and applications of these results to the task determining if solutions exist and finding them when they do!. This includes homework, boardwork and Excursions. Exercises will be assigned daily and followed up by timely feedback.

Assessment: Evaluation of written and oral presentations for proper use of proofs techniques from your first course in differential equations, construction of proofs for results related to systems of differential equations and use of these results to determine if solutions and exist find solutions (of systems of differential equations) when they do!

1. Construct (as well as recreate) formal proofs of propositions and theorems that address concepts discussed during the course of the semester.

Skill Development: Individual, small group and whole group constructions of logical and valid proofs of theorems and propositions. This includes homework, boardwork and Excursions. Exercises will be assigned daily and followed up by timely feedback.

Assessment: Evaluations of written and oral presentations of proofs for proper logic and validity.

1. Present your work clearly and concisely in both written and oral form. Organization and logical flow will be the secrets to success in meeting this objective.

Skill Development: Individual, small group and whole group creation of proofs of stated propositions. Daily exercises with review the following day.

Assessment: Evaluation of all written assignments and oral presentations by both professor and colleagues for clarity and concise language. This includes homework, boardwork and Excursions.

1. Learning Outcome: Program and run the differential analyzer to solve a variety of differential equations.

*Skill Development:* Small group and whole group laboratory analysis of how the mechanics of the differential analyzer models mathematics and hands-on setup of particular models on the machine. These studies will take place in the Marshall Differential Analyzer Lab. Lab experience supported and enhanced by formal written lab exercises.

*Assessment:* Evaluation of lab reports for proper descriptions of the required programming and the associated output of the differential analyzer and analysis of what information the output offers us.

1. Learning Outcome: Construct at least two different valid and logical approaches to a given problem.

*Skill Development:* Individual, small group and whole group discussions with peers and presentations for peers of multiple approaches to the same problem. Exercises assigned daily followed by timely feedback.

*Assessment:* Evaluation of solutions for the proper use of more than one approach to an exercise presented in both written and oral forms. This includes homework, boardwork and Excursions.

This will be a spiritual (and spirited) study of differential equations.

**Textbook:** The Theory of Differential Equations – Classical and Qualitative

Walter G. Kelley and Allan C. Peterson

Springer

The book begins with a discussion of qualitative behavior of solutions of first order equations. The topics include existence and uniqueness of solutions, equilibria, stability and bifurcations. Next they move to systems of equations and offer a similar study of properties of solutions. Read the material carefully and more than once. Create your own mental picture of what is being described. Look at it from your own perspective. I have found this to be the best way to understand sometimes complex ideas.

**Grading Procedure:** You grade will be calculated using the following percentages:

Homework: 15%

Boardwork: 15%

Midterm Exam: 35%

Final Exam: 35%

I want contribute to your quest to be a lifelong learners. To achieve this goal, I have planned the following format for our class time.

On Thursdays I will give you a brief overview of the material I would like you to study and give you a set of exercises to work on.

On Tuesdays you will make presentations at the board of the work you have done on the exercises.

There will be two tests during the semester, a midterm and a final exam **Thursday, May 9, 10:15 A.M. – 12:15 P.M**. In the event you are not able to take the exam on the scheduled date because of very serious circumstances, (see Graduate Catalog, pp. 47-48, for the list of excused absences) please contact me before the scheduled exam time so that we can plan a time for you to take the exam early.

Your final grade will be determined using the following scale:

90% - 100% A

80% - 89% B

70% - 79% C

60% - 69% D

0% - 59% F

My best advice (It’s free!) is for you to keep up with your reading and homework assignments.

**Attendance Policy:** I expect you to be in class every day you are physically able. It is your responsibility to determine what you missed in the event you are unable to attend class. Requesting notes from a colleague would be wise. I am happy to give you information about any assignments you missed. If you miss an exam or a deadline for an assignment and your absence is excused (See University Attendance Policy, page 1 of this document), you have one week after the date of the excused absence to make it up.

**Have a great semester and let me know if I can help you.**

**If I can’t answer your question, I’ll find someone who will!**

**Cheers!**

**Dr. Lawrence**

A Schedule of Events for Advanced Differential Equations

Week 1 Defining Solutions of Differential Equations

January 14, 16 Discussions of Existence and Uniqueness

Boardwork

Week 2 A Review of Solution Techniques for Particular

January 20, 22 Classes of Differential Equations

The Logistic Equation

Examples and Proofs of Lovely Theorems

Boardwork

Week 3 Bifurcation Theory

January 25, 27 Examples and Proofs of Lovely Theorems

Boardwork

Week 4 An Introduction to Linear Systems

February 3, 5 and the Linear Vector Differential Equations

Examples and Proofs of Lovely Theorems

Boardwork

Week 5 Linearly Independent Solutions of the Linea

February 10, 12 Vector Differential Equations

Eigenvalues and Eigenvectors

Examples and Proofs of Lovely Theorems

Boardwork

Week 6 The Matrix Differential Equation –

February 17, 19 Solutions and Existence and Uniqueness

Examples and Proofs of Lovely Theorems

Boardwork

Week 7 The Fundamental Solution of the Homogeneous

February 24, 26 Linear Vector Differential Equation

Examples and Proofs of Lovely Theorems

Week 8 The Matrix Exponential Function

March 3,5 Examples and Proofs of Lovely Theorems

Mid-Term Exam

Week 9 Properties of the Matrix Exponential Function

March 10, 12 The Variation of Constants Formula

Examples and Proofs of Lovely Theorems

Week 10 Spring Break! Enjoy!!

March 16 - 20

Week 11 A Stability Theorem

March 24, 26 Induced Matrix Norms

Examples and Proofs of Lovely Theorems

Week 12 The Lozinski Measure

March 31, April 2 and its Properties

Stability of the Trivial Solution

Examples and Proofs of Lovely Theorems

Week 13 An Introduction to Autonomous

April 7, 9 Systems

Examples and Proofs of Lovely Theorems

Week 14 Phase Plane Diagrams

April 14, 16 for the Second Order System

Examples and Proofs of Lovely Theorems

Week 15 Stability of Nonlinear Systems

April 21, 23 Examples and Proofs of Lovely Theorems

Week 16 Perturbations Methods

April 28, 30 Periodic Solutions

Final Exam: Thursday, May 7, 10:15 – 12:15